Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3
Gold Level (Harder) G3

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
62	54	46	37	30	22

1. Given that

$$\frac{2x^4 - 3x^2 + x + 1}{(x^2 - 1)} \equiv (ax^2 + bx + c) + \frac{dx + e}{(x^2 - 1)},$$

find the values of the constants a, b, c, d and e.

(4)

January 2008

2.

$$\text{f}(x) = 5 \cos x + 12 \sin x.$$

Given that $\text{f}(x) = R \cos (x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$,

(*a*) find the value of R and the value of α to 3 decimal places.

(4)

(*b*) Hence solve the equation

$$5 \cos x + 12 \sin x = 6$$

for $0 \leq x < 2\pi$.

(5)

(*c*) (i) Write down the maximum value of $5 \cos x + 12 \sin x$.

(1)

(ii) Find the smallest positive value of x for which this maximum value occurs.

(2)

June 2008

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3. Find all the solutions of

$$2 \cos 2\theta = 1 - 2 \sin \theta$$

in the interval $0 \leq \theta < 360°$.

(6)

January 2011

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4. (*a*) Express 6 cos θ + 8 sin θ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 decimal places.

(4)

(*b*)

$$\mathrm{p}(\theta) = \frac{4}{12 + 6\cos\theta + 8\sin\theta}, \quad 0 \leq \theta \leq 2\pi.$$

Calculate

(i) the maximum value of $\mathrm{p}(\theta)$,

(ii) the value of θ at which the maximum occurs.

(4)

January 2013

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5. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + \ln x, \quad x > 0, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

(*a*) Write down the range of g.

(1)

(*b*) Show that the composite function fg is defined by

$$\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}, \quad x \in \mathbb{R}.$$

(2)

(*c*) Write down the range of fg.

(1)

(*d*) Solve the equation $\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{fg}(x)] = x(x\mathrm{e}^{x^2} + 2)$.

(6)

January 2009

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6. The function f is defined by

$$\mathrm{f}: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5.$$

(*a*) Find $\mathrm{f}^{-1}(x)$.

(3)

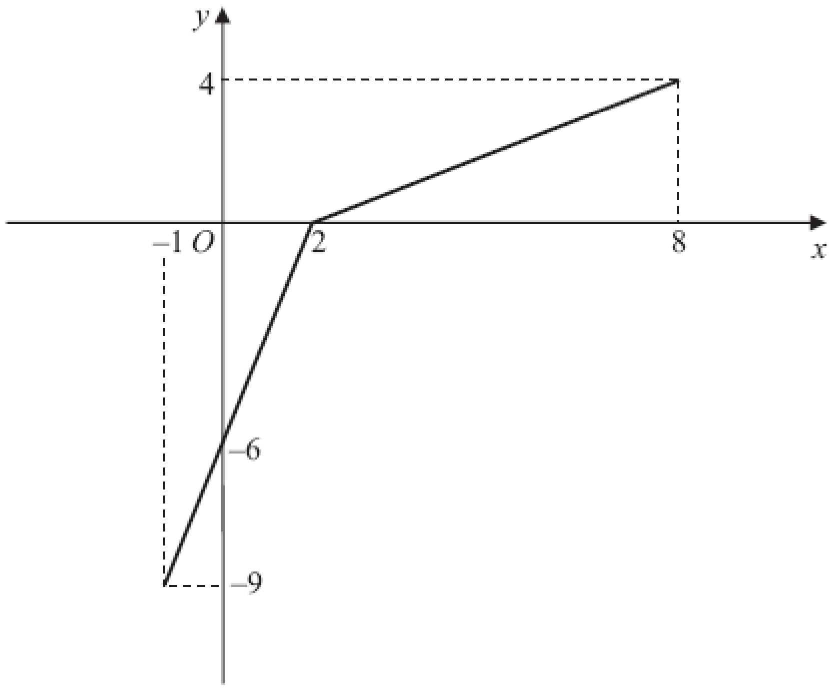

Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = \mathrm{g}(x)$.

(*b*) Write down the range of g.

(1)

(*c*) Find gg(2).

(2)

(*d*) Find fg(8).

(2)

(*e*) On separate diagrams, sketch the graph with equation

(i) $y = |\mathrm{g}(x)|$,

(ii) $y = \mathrm{g}^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(*f*) State the domain of the inverse function g^{-1}.

(1)

January 2011

7. A curve C has equation

$$y = 3\sin 2x + 4\cos 2x, \qquad -\pi \leq x \leq \pi\,.$$

The point $A(0, 4)$ lies on C.

(*a*) Find an equation of the normal to the curve C at A.

(5)

(*b*) Express y in the form $R\sin(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the value of α to 3 significant figures.

(4)

(*c*) Find the coordinates of the points of intersection of the curve C with the x-axis.
Give your answers to 2 decimal places.

(4)

January 2008

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8. The value of Bob's car can be calculated from the formula

$$V = 17000\mathrm{e}^{-0.25t} + 2000\mathrm{e}^{-0.5t} + 500.$$

where V is the value of the car in pounds (£) and t is the age in years.

(*a*) Find the value of the car when $t = 0$.

(1)

(*b*) Calculate the exact value of t when $V = 9500$.

(4)

(*c*) Find the rate at which the value of the car is decreasing at the instant when $t = 8$.
Give your answer in pounds per year to the nearest pound.

(4)

January 2013

TOTAL FOR PAPER: 75 MARKS

END

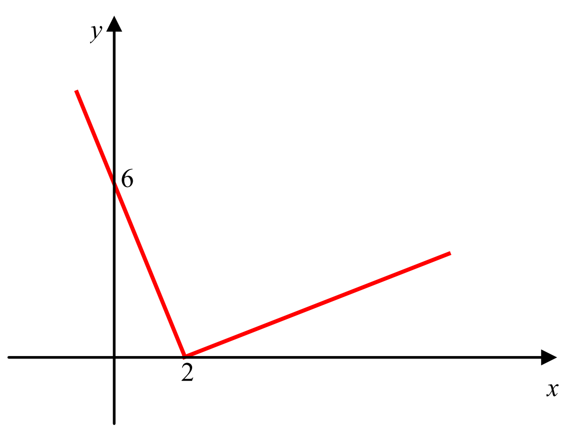
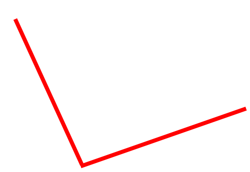
Question Number	Scheme	Marks
1.	$$\begin{array}{r|l} & 2x^2 \quad -1 \\ \hline x^2-1 & 2x^4 \quad -3x^2+x+1 \\ & 2x^4 \quad -2x^2 \\ \hline & \quad -x^2+x+1 \\ & \quad -x^2 \quad +1 \\ \hline & \quad x \end{array}$$	M1
	$a=2$ stated or implied	A1
	$c=-1$ stated or implied	A1
	$2x^2-1+\dfrac{x}{x^2-1}$	
	$a=2, b=0, c=-1, d=1, e=0$	
	$d=1$ and $b=0, e=0$ stated or implied	A1
		[4]

2. (*a*)	$R^2=5^2+12^2$	M1
	$R=13$	A1
	$\tan\alpha=\dfrac{12}{5}$	M1
	$\alpha\approx 1.176$	A1 cao (4)
(*b*)	$\cos(x-\alpha)=\dfrac{6}{13}$	M1
	$x-\alpha=\arccos\dfrac{6}{13}=1.091\ \ldots$	A1
	$x=1.091\ \ldots+1.176\ \ldots\approx 2.267\ldots$ awrt 2.3	A1
	$x-\alpha=-1.091\ \ldots$ accept … = 5.19 … for M	M1
	$x=-1.091\ \ldots+1.176\ \ldots\approx 0.0849\ldots$ awrt 0.084 or 0.085	A1 (5)
(*c*)(i)	$R_{\max}=13$ ft their R	B1 ft
(ii)	At the maximum, $\cos(x-\alpha)=1$ *or* $x-\alpha=0$	M1
	$x=\alpha=1.176\ \ldots$ awrt 1.2, ft their α	A1ft (3)
		(12 marks)

Question Number	Scheme		Marks
3.	$2\cos 2\theta = 1 - 2\sin\theta$		
	$2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$	Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$.	M1
	$2 - 4\sin^2\theta = 1 - 2\sin\theta$		
	$4\sin^2\theta - 2\sin\theta - 1 = 0$	Forms a "quadratic in sine" = 0	M1(*)
	$\sin\theta = \frac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$	Applies the quadratic formula See notes for alternative methods.	M1
	PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$		
		Any one correct answer	A1
	$\theta = \{54, 126, 198, 342\}$	180-their pv	dM1(*)
		All four solutions correct.	A1
			[6]

Question Number	Scheme	Marks
4.	(a) $R^2 = 6^2 + 8^2 \Rightarrow R = 10$	M1A1
	$\tan\alpha = \frac{8}{6} \Rightarrow \alpha = \text{awrt } 0.927$	M1A1
		(4)
	(b)(i) $\mathrm{p}(x) = \frac{4}{12 + 10\cos(\theta - 0.927)}$	
	$\mathrm{p}(x) = \frac{4}{12 - 10}$	M1
	Maximum = 2	A1
		(2)
	(b)(ii) $\theta - \text{'}their\ \alpha\text{'} = \pi$	M1
	$\theta = \text{awrt } 4.07$	A1 (2)
		(8 marks)

Question Number	Scheme	Marks
5 (a)	$\mathrm{g}(x) \geq 1$	B1 (1)
(b)	$\mathrm{fg}(x) = \mathrm{f}\left(\mathrm{e}^{x^2}\right) = 3\mathrm{e}^{x^2} + \ln \mathrm{e}^{x^2}$	M1
	$= x^2 + 3\mathrm{e}^{x^2}$ ∗	A1 (2)
	$\left(\mathrm{fg} : x \mapsto x^2 + 3\mathrm{e}^{x^2}\right)$	
(c)	$\mathrm{fg}(x) \geq 3$	B1 (1)
(d)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2 + 3\mathrm{e}^{x^2}\right) = 2x + 6x\mathrm{e}^{x^2}$	M1 A1
	$2x + 6x\mathrm{e}^{x^2} = x^2\mathrm{e}^{x^2} + 2x$	
	$\mathrm{e}^{x^2}\left(6x - x^2\right) = 0$	M1
	$\mathrm{e}^{x^2} \neq 0$, $6x - x^2 = 0$	A1
	$x = 0, 6$	A1 A1 (6)
		[10]

Question Number	Scheme	Marks
6. **(a)**	$y = \dfrac{3-2x}{x-5} \Rightarrow y(x-5) = 3-2x$ Attempt to make x (or swapped y) the subject $xy - 5y = 3 - 2x$ $\Rightarrow xy + 2x = 3 + 5y \Rightarrow x(y+2) = 3+5y$ Collect x terms together and factorise. $\Rightarrow x = \dfrac{3+5y}{y+2} \quad \therefore \underline{\mathrm{f}^{-1}(x) = \dfrac{3+5x}{x+2}}$ $\underline{\dfrac{3+5x}{x+2}}$	M1 M1 A1 oe (3)
(b)	Range of g is $-9 \le \mathrm{g}(x) \le 4$ or $-9 \le y \le 4$ <u>Correct Range</u>	B1 (1)
(c)	Deduces that $\mathrm{g}(2)$ is 0. Seen or implied. $\mathrm{g\,g}(2) = \mathrm{g}(0) = -6$, from sketch. -6	M1 A1 (2)
(d)	$\mathrm{fg}(8) = \mathrm{f}(4)$ Correct order g followed by f $= \dfrac{3-4(2)}{4-5} = \dfrac{-5}{-1} = \underline{5}$ 5	M1 A1 (2)
(e)(i)	Correct shape   $(2, \{0\}), (\{0\}, 6)$	B1 B1

Question Number	Scheme	Marks
7.	(a) $\dfrac{dy}{dx} = 6\cos 2x - 8\sin 2x$	M1 A1
	$\left(\dfrac{dy}{dx}\right)_0 = 6$	B1
	$y - 4 = -\dfrac{1}{6}x$ or equivalent	M1 A1 (5)
	(b) $R = \sqrt{\left(3^2 + 4^2\right)} = 5$	M1 A1
	$\tan\alpha = \dfrac{4}{3},\ \alpha \approx 0.927$ awrt 0.927	M1 A1 (4)
	(c) $\sin\left(2x + \text{ their } \alpha\right) = 0$	M1
	$x = -2.03, -0.46, 1.11, 2.68$	A1 A1 A1 (4)
	First A1 any correct solution; second A1 a second correct solution; third A1 all four correct and to the specified accuracy or better. Ignore the y-coordinate.	**[13]**

Question Number	Scheme	Marks
8.	(a) (£) 19500	B1
		(1)
	(b) $9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$	
	$17e^{-0.25t} + 2e^{-0.5t} = 9$	
	$(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$	
	$0 = 9e^{0.5t} - 17e^{0.25t} - 2$	M1
	$0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
	$e^{0.25t} = 2$	A1
	$t = 4\ln(2)$ *oe*	A1
		(4)
	(c) $(\frac{\mathrm{d}V}{\mathrm{d}t}) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
	When t=8 Decrease = 593 (£/year)	M1A1
		(4)
		(9 marks)

6665 Core Mathematics C3 – G2 Mark Scheme

Comments from Examiners' Reports:

1. A large number of candidates did not find this a friendly start to the paper, with quite a high proportion attempting the question more than once. There were many who dividing by $x^2 - 1$, showed insufficient knowledge of the method, stopping their long division before the final subtraction. Those getting as far as a linear remainder usually obtained the correct values of *a* and *c*, but the remainder was often incorrect. Errors often arose when not using the strategy of replacing $2x^4 - 3x^2 + x + 1$ by $2x^4 + 0x^3 - 3x^2 + x + 1$ and $x^2 - 1$ by $x^2 + 0x - 1$. It was not unusual to see candidates who completed long division correctly but who then, apparently not recognising the relevance of this to the question, went on to try other methods.

Most of those who used methods of equating coefficients and substituting values found 5 independent equations but completely correct solutions using these methods were uncommon. Very few decomposed the numerator and, generally, these appeared to be strong candidates. Of those who attempted to divide first by $x + 1$, and then by $x - 1$, few were able to deal with the remainders in a correct way.

2. This question was a good source of marks for the majority of candidates. The method needed in part (a) was well understood, although many candidates failed to recognise the need to give the angle to 3 decimal places and in radians. The condition in the question $0 < \alpha < \frac{\pi}{2}$ implies that α is in radians. Although penalised here, degrees were accepted in the other parts of the question.

Part (b) produced many good solutions with many scoring 3 or 4 marks out of 5. Only the best candidates, however, were able to produce a second solution within range. Commonly a second solution greater than 2π was found or the second solution was ignored altogether. Most saw the connection between part (a) and (c) and there were few attempts to use differentiation. A few thought that the maximum of $\mathrm{f}(x)$ was 5 + 12 and that $\cos(x - \alpha)$ had a maximum at $x - a = \frac{\pi}{2}$ or π.

3. This question was attempted by most candidates but was not answered that well. Only the best candidates produce fully correct solutions. Most understood that $\cos 2x$ should be replaced, although $1 - \sin^2 x$ was sometimes seen instead of $1 - 2\sin^2 x$. The use of brackets was careless in some cases. Many used $\cos^2 x - \sin^2 x$ first, which was acceptable as long as the $\cos^2 x$ was replaced subsequently by $1 - \sin^2 x$. The majority of good candidates did arrive at the correct quadratic equation but solving it was one of the least successful parts of this paper. Many candidates could not believe that it would not factorise! Often several attempts were made before moving on, some just gave up. Those using the quadratic formula did not always quote it correctly. Others made errors in the substitution. A minority used 'completing the square' to solve, though with mixed success. A few more successfully used equation solver on their calculators. If they got this far, one or two angles were found, but many didn't find all four. They either rejected the negative value or only gave one answer for it within the range.

4. In Q4(a) candidates demonstrated knowledge of using the R cos (θ – α) identity and were generally successful in finding both R and α. Most candidates were finding R and α independently of each other using division and Pythagoras' theorem. Most candidates gave α to the required number of decimal places; it was rare to see answers rounded to 2 decimal places or more. Most candidates also gave their answer for α in radians, but 53.1 was also seen. Candidates who found α first were more likely to then use the numbers 3 and 4 (incorrectly, instead of 6 and 8) in Pythagoras' theorem to determine R.

In Q4(b) very few realised that the maximum value of p was achieved using the minimum value of the denominator. However those who did realise this, gained the marks in Q4(b)(i) and generally went on to achieve full marks in Q4(b)(ii) as well. The majority of candidates thought they were looking for the maximum value of the denominator, setting cos (θ – α) = 1, leading to $\frac{2}{11}$. Calculus could have, and was occasionally, used but almost invariably led to incorrect solutions.

5. Parts (a) and (c) were rarely correct. Relatively few candidates showed an understanding of the concept that the range of a function is the possible set of values of $\mathrm{g}(x)$ or $\mathrm{fg}(x)$ and those who did often failed to discriminate between "greater than 1" and "greater than or equal to 1". Part (b) was generally very well done and the confusion between $\mathrm{gf}(x)$ and $\mathrm{fg}(x)$ was rarely seen.

Part (d) proved very discriminating and many did not realise that they were being asked to solve an equation. Some thought that they were being asked to prove that $\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{fg}(x)\right] = x\left(x\mathrm{e}^{x^2}+2\right)$ and they could gain the first two marks. However those who started by differentiating $x\left(x\mathrm{e}^{x^2}+2\right)$ could gain no credit. Those who understood the question correctly often had difficulties in applying the chain rule to $3\mathrm{e}^{x^2}$. When the correct equation $2x+6x\mathrm{e}^{x^2} = x\left(x\mathrm{e}^{x^2}+2\right)$ was obtained and this simplified to $6x\mathrm{e}^{x^2} = x^2\mathrm{e}^{x^2}$, candidates often had problems with the e^{x^2} term not realising that, as e^{x^2} cannot be 0, it can be cancelled. Even strong candidates often omitted the solution $x = 0$ and full marks were rarely obtained on part (d).

6. In part (a) it is worth noting that a number of candidates were weak on notation with a significant number finding the derivative $\mathrm{f}'(x)$ rather than the inverse function $\mathrm{f}^{-1}(x)$. Those who tried to find the inverse were generally successful, although a worryingly large minority found the algebraic manipulation beyond them.

Many candidates gave the correct answer in part (b) which could be easily found from the graph. A few used domain notation rather than the range.

Parts (c) and (d) were often not attempted. Of those that did, many failed to see that g(2) and g(8) could be read from the graph, and instead worked out the two linear equations for the function g. This could lead to the correct solution but rarely did they were not always correctly applied. A popular incorrect solution involved finding g(2) correctly, but then simply squaring to get gg(2) = g(2) × g(2) = 0. Part (d) was generally more successful as the function f(x) was given.

In part (e) accurate sketch graphs were usually seen in (i), with candidates generally familiar with the idea of a modulus. Incorrect or missing co-ordinates lead to the loss of some marks. There was less success in (ii) with the sketch of the inverse function. Many were able to remember to reflect in the line $y = x$ but there were many incorrect attempts, again with missing or incorrect co-ordinates

In part (f) a substantial majority realised that the domain of the inverse was the same as the range of the original function, but there was again some confusion about which variable 'x' or 'y' should be used.

7. Not available!

8. Q8(a) was usually correct, although a few candidates thought that e^0 was 0.

Q8(b) proved to be the most challenging part of the paper. Most candidates knew how to start, correctly equating to 9500 but the majority did not collect the terms together on one side and applied the laws of logs incorrectly resulting in incorrect equations, gaining no marks at all. Those that managed to get the correct quadratic usually went on to gain full marks.

Q8(c) had many correct responses. Many realised that differentiation was required, although a number of candidates substituted 8 into the original equation and some tried to differentiate after making this substitution. There were errors in differentiating with an extra t appearing. A common incorrect method was to calculate the value for 2 consecutive years, usually 8 and 9 or 7 and 8, then subtract to find the yearly rate.

Statistics for C3 Practice Paper G3

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4		58	2.30		3.58	2.87	2.42	2.08	1.86	1.21
2	7		67	8.07		10.54	8.83	7.25	5.30	3.31	1.41
3	9		57	3.43	5.82	4.95	3.96	3.26	2.31	1.47	0.72
4	7	4	56	4.44	7.28	5.21	4.49	3.99	3.47	3.03	1.67
5	8		49	4.89		6.78	5.40	4.30	3.37	2.48	1.70
6	10		60	7.80	12.06	10.10	7.97	6.42	4.93	3.77	2.13
7	12		60	7.81		11.57	9.20	7.41	5.37	3.64	1.28
8	7	1	41	3.72	7.97	5.59	4.29	3.35	2.69	1.71	1.10
	75		**57**	**42.46**		**58.32**	**47.01**	**38.40**	**29.52**	**21.27**	**11.22**